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A Pinching Theorem for Totally Real Riemannian Foliations with Parallelized Mean Curvature Vectors on a Quaternion Projective Space

PENG Hui-chun¹, LI Zhi-bo²

(1- Department of Mathematics and Physics, North China Electric Power University, Beijing 102206;

2- Department of Mathematics, Zhengzhou University, Zhengzhou 450052)

Abstract: By utilizing the divergence evaluation method of Nakagawa and Takagi for Harmonic foliations on the sphere, the divergence of a vector field on a totally real Riemannian foliation with parallelized mean curvature vectors on a quaternion projective space is found out. By pinching the length of the second fundamental form, a formula of Simons' type is obtained. The geometric restrictions of the length of the second fundamental form are proposed so that each leave of foliation is assumed to be totally umbilical.

Keywords: Riemannian foliations; quaternion projective space; mean curvature; divergence; totally umbilical

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1 Introduction

Geometric notions in the theory of Riemannian submanifolds have their counterparts for foliations on Riemannian manifolds. The harmonic foliations on Riemannian manifolds have been extensively studied in recent years^[1-4]. Many harmonic foliations which are not totally geodesic are known. It has been known that under some geometric restrictions, harmonicity implies totally geodesicness. But the geometric property of Riemannian foliations with parallelized mean curvature vectors in spaces are still unknown. The purpose of this paper is to study the totally real Riemannian foliations with parallelized mean curvature vectors on a quaternion projective space. Improving the method of Nakagawa and Takagi^[3], we calculate divergence of the vector field and obtained a formula of Simons'. Then We prove that the totally real Riemannian foliations with parallelized mean curvature vectors on a quaternion projective space satisfying the length square of second fundamental form

$$S \leq \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\}, \quad c > 0, \quad n > 2,$$

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Biography: Peng Huichun (Born in 1971), Female, Master, Associate Professor. Research field: differential geometry.

is totally umbilical, or

$$S = \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\}.$$

2 Preliminaries

Let $QP^n(c)$ be a $4n$ -dimensional ($n > 2$) quaternion projective space with constant quaternion sectional curvature $c > 0$. A submanifold M is said to be a totally real submanifold of $QP^n(c)$ if every tangent space of M is mapped into its totally real normal space by the quaternion structures of $QP^n(c)$. \mathbf{F} is said to be totally real foliation in $QP^n(c)$, if each leaf \mathbf{L} of \mathbf{F} is a totally real submanifold in $QP^n(c)$. Let \mathbf{F} be a totally real n -dimensional foliation on $QP^n(c)$. Considering \mathbf{F} as a integrable distribution on $QP^n(c)$, we denote the orthogonal distribution of \mathbf{F} by \mathbf{F}^\perp , which is called the normal plane field. For any vector field X on $QP^n(c)$, we decompose it as $X = X' + X''$, where X' (resp. X'') is tangent (resp. normal) to \mathbf{F} . We define two tensors A and h of type $(1,2)$ on $QP^n(c)$ by

$$A(X, Y) = -(\nabla_{Y''} X'')', \quad h(X, Y) = -(\nabla_{Y'} X')'', \quad (1)$$

for any vector fields X and Y on $QP^n(c)$, where ∇ denotes the Riemannian connection with respect to the Riemannian metric g of $QP^n(c)$.

The restriction of h to each leaf of \mathbf{F} is called the second fundamental form of the leaf. We define the second fundamental form B of the normal field \mathbf{F}^\perp by^[2]

$$B(X, Y) = \frac{1}{2} \{A(X, Y) + A(Y, X)\}, \quad (2)$$

for any vector fields X and Y on $QP^n(c)$. We will use throughout this paper, the following convention on the range of indices unless otherwise stated:

$$\begin{aligned} A, B, C, \dots &= 1, \dots, n, & I(1), \dots, I(n), & J(1), \dots, J(n), \\ & & K(1), \dots, K(n); & i, j, k, \dots = 1, 2, \dots, n; \\ \alpha, \beta, \gamma, \dots &= I(1), \dots, I(n), & J(1), \dots, J(n), & K(1), \dots, K(n). \end{aligned}$$

Let $e_1, \dots, e_n, e_{I(1)}, \dots, e_{I(n)}, e_{J(1)}, \dots, e_{J(n)}, e_{K(1)}, \dots, e_{K(n)}$ be a locally defined orthonormal frame field of $QP^n(c)$ such that, restricting to \mathbf{F} , e_1, \dots, e_n are tangent to \mathbf{F} . Let $\{\omega_A\}$ be the dual frame field. The structure equations of $QP^n(c)$ are given as follows^[5]:

$$d\omega_A = -\omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \quad (3)$$

$$d\omega_{AB} = -\omega_{AC} \wedge \omega_{CB} + \frac{1}{2} K_{ABCD} \omega_C \wedge \omega_D, \quad (4)$$

$$\begin{aligned} K_{ABCD} = \frac{C}{4} (\delta_{AC} \delta_{BD} - \delta_{AD} \delta_{BC} + I_{AC} I_{BD} - I_{AD} I_{BC} + 2I_{AB} I_{CD} + J_{AC} J_{BD} \\ - J_{AD} J_{BC} + 2J_{AB} J_{CD} + K_{AC} K_{BD} - K_{AD} K_{BC} + 2K_{AB} J_{CD}), \end{aligned} \quad (5)$$

where I, J and K are quaternion structures of $QP^n(c)$, and K_{ABCD} is the curvature tensor of $QP^n(c)$. Restrict to \mathbf{F} :

$$\omega_\alpha = 0, \quad \omega_{ij} = \omega_{\alpha\beta}, \quad \omega_{\alpha i} = \omega_{i\alpha}, \quad (6)$$

$$\omega_{\alpha i} = \sum h_{ij}^\alpha \omega_j + \sum A_{\alpha\beta}^j \omega_\beta, \quad h_{ij}^\alpha = h_{ji}^\alpha = h_{jk}^\beta = h_{ik}^\beta, \quad (7)$$

$$d\omega_i = -\omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \quad (8)$$

$$d\omega_{ij} = -\omega_{ik} \wedge \omega_{kj} + \frac{1}{2} R_{ijkl} \omega_k \wedge \omega_l, \quad (9)$$

$$d\omega_{\alpha\beta} = -\omega_{\alpha\gamma} \wedge \omega_{\gamma\beta} + \frac{1}{2} R_{\alpha\beta kl} \omega_k \wedge \omega_l, \quad (10)$$

where R_{ijkl} and $R_{\alpha\beta kl}$ denote the curvature tensor of tangent connection and normal connection of \mathbf{F} , respectively. The Riemannian connection ∇ on $QP^n(c)$ is given by $\nabla_{e_A} e_B = \sum \omega_{CB}(e_A) e_C$. Then the components h_{BC}^A (resp. A_{CD}^B) of h (resp. A) with respect to $\{e_A\}$ and $\{\omega_A\}$ are given by

$$h_{ij}^\alpha = \omega_{\alpha i}(e_j), \quad A_{\alpha\beta}^i = \omega_{\alpha i}(e_\beta). \quad (11)$$

The second fundamental form \mathbf{H} of \mathbf{F} is: $\mathbf{H} = \sum_{a,i,j} h_{ij}^\alpha \omega_i \omega_j e_\alpha$. The length square of \mathbf{H} is: $S = \sum_{a,i,j} (h_{ij}^\alpha)^2$. For each α , H^α denotes the matrix (h_{ij}^α) . $\xi = \frac{1}{n} \sum_\alpha (\text{tr} H^\alpha) e_\alpha$ is called the mean curvature vector, and $H = \|\xi\|$ is called the mean curvature, where tr denotes the trace of the matrix (h_{ij}^α) . The foliation \mathbf{F} is said to be harmonic or minimal (resp. totally geodesic) if $\sum h_{jj}^\alpha = 0$ (resp. $h_{jj}^\alpha = 0$). The normal plane field \mathbf{F}^\perp is said to be minimal if $\text{tr} B = \sum A_{\alpha\alpha}^i e_i = 0$. The normal plane field \mathbf{F}^\perp is said to be totally geodesic, if $B = 0$. The metric is bundle-like if and only if $A_{\alpha\beta}^i = -A_{\beta\alpha}^i$, which imply $B = 0$. The foliations \mathbf{F} with bundle-like metric is called Riemannian foliations. By (5), we have

$$R_{ijkl} = K_{ijkl} + \sum_\alpha (h_{ik}^\alpha h_{jl}^\alpha - h_{il}^\alpha h_{jk}^\alpha), \quad R_{\alpha\beta kl} = K_{\alpha\beta kl} + \sum_m (h_{km}^\alpha h_{ml}^\beta - h_{km}^\beta h_{ml}^\alpha). \quad (12)$$

For a tensor field $T = (T_{B_1 \dots B_s}^{A_1 \dots A_r})$ on $QP^n(c)$, we define its 1-order covariant derivatives by^[9]

$$\begin{aligned} T_{B_1 \dots B_s C}^{A_1 \dots A_r} \omega_C &= dT_{B_1 \dots B_s}^{A_1 \dots A_r} - \sum T_{B_1 \dots B_s}^{A_1 \dots A_{a-1}, C, A_a+1 \dots A_r} \omega_{CA_a} \\ &\quad - \sum T_{B_1 \dots B_{b-1}, C, B_b+1 \dots B_s}^{A_1 \dots A_r} \omega_{CB_b}. \end{aligned} \quad (13)$$

Then we have the definitions of (h_{BCD}^A) and (A_{BCD}^A) . The detailed formulae can be found in reference [2].

3 Calculus of the divergence

A vector field $v = \sum \nu_A e_A$ on $QP^n(c)$ is defined by

$$\nu_k = \sum h_{ij}^\alpha h_{ijk}^\alpha, \quad \nu_\alpha = 0. \quad (14)$$

By [6], we know that the divergence of the vector field v is defined by

$$\delta v = \text{div} v = \sum \nu_{AA} = \sum \nu_{kk} + \sum \nu_{\alpha\alpha}. \quad (15)$$

Since \mathbf{F} is a Riemannian foliations, i.e., $A_{\alpha\beta}^i = -A_{\beta\alpha}^i$, then

$$A_{\alpha\alpha}^i = 0. \quad (16)$$

Taking exterior differentiation of (14) and using (16), we have

$$\begin{aligned} \nu_{kk} &= \sum h_{ij}^\alpha h_{ijk}^\alpha + \sum h_{ij}^\alpha h_{ijkk}^\alpha + \sum h_{ij}^\alpha h_{ij}^\beta h_{mk}^\beta h_{mk}^\alpha \\ &\quad + \sum h_{ij}^\alpha h_{jl}^\alpha h_{lk}^\beta h_{kj}^\beta + \sum h_{ij}^\alpha h_{jl}^\beta h_{lk}^\beta h_{ki}^\beta + \sum h_{ij}^\alpha h_{ij\beta}^\beta h_{kk}^\beta, \\ \nu_{\alpha\alpha} &= \sum_{k,\alpha} \nu_k A_{\alpha\alpha}^k = 0. \end{aligned} \quad (17)$$

It have been obtained that^[2,6]

$$\begin{aligned} h_{ij}^\alpha h_{ijkk}^\alpha &= \sum h_{ij}^\alpha h_{kkij}^\alpha - 2 \sum h_{ij}^\alpha h_{ij}^\beta h_{lk}^\alpha h_{lk}^\beta + 4 \sum h_{ij}^\alpha h_{jl}^\beta h_{lk}^\alpha h_{ki}^\beta \\ &\quad - 2 \sum h_{ij}^\alpha h_{jl}^\alpha h_{lk}^\beta h_{ki}^\beta - 2 \sum h_{ij}^\alpha h_{jk}^\beta h_{kl}^\beta h_{li}^\alpha + 2 \sum h_{ij}^\alpha h_{jl}^\beta h_{li}^\alpha h_{kk}^\beta \\ &\quad - \sum h_{ij}^\alpha h_{ij\beta}^\alpha h_{kk}^\alpha + \sum h_{ij}^\alpha h_{kk\beta}^\alpha h_{ij}^\beta - c \sum_\alpha \left(\sum_k h_{kk}^\alpha \right)^2 + cn \sum_{\alpha,i,j} (h_{ij}^\alpha)^2. \end{aligned} \quad (18)$$

Now assume e_α to be the mean curvature vector. Hence $\sum_k h_{kk}^\beta = 0$ ($\beta \neq \alpha$). From (13), we have

$$\begin{cases} \sum_{k,c} h_{kkc}^\alpha \omega_c = ndH, \\ \sum_{k,c} h_{kkc}^\beta \omega_c = nH\omega_{\beta\alpha}, \quad \beta \neq \alpha. \end{cases} \quad (19)$$

Here H is the mean curvature. The vector e_α is parallel in the normal bundle.

According to (19), we have

$$\begin{aligned} h_{ij}^\alpha h_{ijkk}^\alpha &= -2 \sum h_{ij}^\alpha h_{ij}^\beta h_{lk}^\alpha h_{lk}^\beta + 2 \sum h_{ij}^\alpha h_{jl}^\beta h_{lk}^\alpha h_{ki}^\beta - 2 \sum h_{ij}^\alpha h_{jl}^\alpha h_{lk}^\beta h_{ki}^\beta \\ &\quad - 2 \sum h_{ij}^\alpha h_{jk}^\beta h_{kl}^\beta h_{li}^\alpha + \sum h_{ij}^\alpha h_{jl}^\beta h_{li}^\alpha h_{kk}^\beta - \sum h_{ij}^\alpha h_{ij\beta}^\alpha h_{kk}^\beta \\ &\quad - c \sum_\alpha \left(\sum_k h_{kk}^\alpha \right)^2 + cn \sum_{\alpha,i,j} (h_{ij}^\alpha)^2. \end{aligned} \quad (20)$$

Let H^α denote the matrix (h_{ij}^α) . The divergence of the vector field v is simplified by

$$\begin{aligned} \delta v &= \sum h_{ijk}^\alpha h_{ijk}^\alpha + \sum_{\alpha,\beta} \text{tr}(H^\alpha H^\beta - H^\beta H^\alpha)(H^\alpha H^\beta - H^\beta H^\alpha) \\ &\quad - \sum_{\alpha,\beta} [\text{tr}(H^\alpha H^\beta)]^2 + \sum_{\alpha,\beta} [\text{tr}(H^\alpha)^2 H^\beta] \text{tr}(H^\beta) \\ &\quad - c \sum_\alpha (\text{tr}(H^\alpha))^2 + cn \sum_\alpha \text{tr}(H^\alpha)^2. \end{aligned} \quad (21)$$

4 Proof of the main theorems

Lemma 4.1^[6] Let A, B be an $n \times n$ symmetric matrix and $N(A) = \text{tr}(AA^T)$. Then

$$N(AB - BA) \leq 2N(A)N(B).$$

Lemma 4.2^[7] There are $2n$ real numbers a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n with $\sum_{i=1}^n b_i = 0$. Then

$$\left(\sum_{i,j} a_i a_j (b_i - b_j)^2 \right)^2 \leq (2n+6) \left(\sum a_i^2 \right)^2 \left(\sum b_j^2 \right)^2.$$

Theorem 4.1 Let $QP^n(c)$ be a $4n$ -dimensional ($n > 2$) quaternion projective space with constant quaternion sectional curvature $c > 0$, and \mathbf{F} be a totally real Riemannian foliation in $QP^n(c)$ with parallelized mean curvature vectors, and the mean curvature be $H(\neq 0)$ with $n \geq 2$. Let e_γ be the mean curvature vector. Hence

$$\int_{QP^n(c)} \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 \left\{ \max \left(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1} \right) S - cn \right\} * 1 \geq 0.$$

Proof Assume that \mathbf{F} is a foliation with parallelized mean curvature vectors and the mean curvature is $H(\neq 0)$. Let $e_\gamma = \xi / \|\xi\|$. Then we have

$$\begin{cases} \text{tr}(H^\alpha) = 0, & \alpha \neq \gamma, \\ \text{tr}(H^\gamma) = nH. \end{cases} \quad (22)$$

From $\omega_{\gamma,\alpha} = 0$, we have

$$H^\gamma H^\alpha = H^\alpha H^\gamma. \quad (23)$$

From (22) and (23), (21) becomes

$$\delta v_\gamma = h_{ijk}^\gamma h_{ijk}^\gamma - \sum_{\beta \neq \gamma} [\text{tr}(H^\gamma H^\beta)]^2 - cn^2 H^2 + cn \text{tr}(H^\gamma)^2, \quad (24)$$

$$\begin{aligned} -\delta v_{(\alpha)} + \sum_{\alpha \neq \gamma} h_{ijk}^\alpha h_{ijk}^\alpha = & - \sum_{\alpha, \beta \neq \gamma} \text{tr}(H^\alpha H^\beta - H^\beta H^\alpha)^2 + \sum_{\alpha, \beta \neq \gamma} [\text{tr}(H^\alpha H^\beta)]^2 \\ & + \sum_{\alpha \neq \gamma} [\text{tr}(H^\alpha H^\gamma)]^2 - \sum_{\alpha \neq \gamma} nH \text{tr}[(H^\alpha)^2 H^\gamma] - cn \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2. \end{aligned} \quad (25)$$

Let

$$I = - \sum_{\alpha, \beta \neq \gamma} \text{tr}(H^\alpha H^\beta - H^\beta H^\alpha)^2 + \sum_{\alpha, \beta \neq \gamma} [\text{tr}(H^\alpha H^\beta)]^2, \quad (26)$$

$$II = \sum_{\alpha \neq \gamma} [\text{tr}(H^\alpha H^\gamma)]^2 - \sum_{\alpha \neq \gamma} nH \text{tr}[(H^\alpha)^2 H^\gamma]. \quad (27)$$

From (26) and Lemma 4.1, we have

$$\begin{aligned} I &= (2(n-1)^2 - (n-1))\sigma_1^2 - (n-1)^2(n-2)(\sigma_1^2 - \sigma_2) \\ &\leq (n-1)(2n-3)\sigma_1^2 \leq \left(\frac{2n-3}{n-1} \right) \left(\sum_{\alpha, \beta \neq \gamma} S_\alpha \right)^2, \end{aligned}$$

where

$$(n-1)\sigma_1 = \sum_{\alpha, \beta \neq \gamma} S_\alpha, \quad \frac{(n-1)(n-2)}{2}\sigma_2 = \sum_{\alpha \neq \beta; \alpha, \beta \neq \gamma} S_\alpha S_\beta.$$

From (27) and for a fixed α , we choose a local field of orthonormal frames $\{e_1, \dots, e_n\}$ such that the matrix (h_{ij}^α) is diagonal, i.e., $h_{ij}^\alpha = 0$ ($i \neq j$). By applying (22) and Lemma 4.2, we obtain

$$\begin{aligned} II &= \left(\sum_i h_{ii}^\gamma h_{ii}^\alpha \right)^2 - \sum_{i,j} h_{ii}^\gamma h_{jj}^\gamma (h_{jj}^\alpha)^2 \\ &= \sum_{i,j} [h_{ii}^\gamma h_{ii}^\alpha h_{jj}^\gamma h_{jj}^\alpha - h_{ii}^\gamma h_{jj}^\gamma (h_{jj}^\alpha)^2] = -\frac{1}{2} \sum_{i,j} h_{ii}^\gamma h_{jj}^\gamma (h_{ii}^\alpha - h_{jj}^\alpha)^2 \\ &\leq \frac{\sqrt{2n+1}}{2} \left(\sum_i h_{ii}^\gamma \right)^2 \left(\sum_j (h_{jj}^\alpha)^2 \right) = \sqrt{\frac{n+3}{2}} \left(\sum_{i,j} (h_{ij}^\gamma)^2 \right) \left(\sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 \right). \end{aligned}$$

Then

$$\begin{aligned} -\delta v_{\alpha \neq \gamma} + \sum_{\alpha \neq \gamma} h_{ijk}^\alpha h_{ijk}^\alpha &\leq \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 \left\{ \sqrt{\frac{n+3}{2}} \sum_{i,j} (h_{ij}^\gamma)^2 + \frac{2n-3}{n-1} \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 - cn \right\} \\ &\leq \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 \left\{ \max \left(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1} \right) S - cn \right\}. \end{aligned}$$

Taking integration on both sides of the above inequality and according to the Stokes theorem, we have

$$\begin{aligned} 0 &\leq \int_{QP^n(c)} \sum_{\alpha \neq \gamma} h_{ijk}^\alpha h_{ijk}^\alpha * 1 \\ &\leq \int_{QP^n(c)} \sum_{\alpha \neq \gamma} (h_{ij}^\alpha)^2 \left\{ \max \left(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1} \right) S - cn \right\} * 1. \end{aligned} \quad (28)$$

The theorem is proved.

Corollary 4.1 Let \mathbf{F} be a totally real Riemannian foliation in $QP^n(c)$ with parallelized mean curvature vectors and the mean curvature be $H(\neq 0)$ with $n \geq 2$. Let e_γ be the mean curvature vector. If the length square of second fundamental form

$$S \leq \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\},$$

then each leaf of \mathbf{F} is totally umbilical, or

$$S = \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\}.$$

Proof Since

$$S \leq \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\},$$

we have

$$\left\{ \max \left(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1} \right) S - cn \right\} \leq 0.$$

From (28), we obtain

$$S = \min \left\{ \frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3} \right\},$$

or

$$\sum_{\alpha \neq \gamma} (h_{ij}^{\alpha})^2 = 0. \quad (29)$$

Let $\mu_{ij} = (h_{ij}^{\gamma} - H\delta_{ij})$ and $U = (\mu_{ij})$. Then $\text{tr}(U^2) = \sum_{i,j} (\mu_{ij})^2 = \text{tr}(H^{\gamma})^2 - nH^2$. From (24) and (29), we have

$$-\delta v_{(\gamma)} + h_{ijk}^{\gamma} h_{ijk}^{\gamma} \leq \text{tr}(H^{\gamma})^2 \sum_{\beta \neq \gamma} \text{tr}(H^{\beta})^2 - cn \text{tr} U^2 = -cn \text{tr} U^2.$$

Taking integration on both sides of the above inequality and using the Stokes theorem, we have

$$0 \leq \int_{QP^n(c)} h_{ijk}^{\gamma} h_{ijk}^{\gamma} * 1 \leq \int_{QP^n(c)} (-cn \text{tr} U^2) * 1.$$

It implies $\text{tr}(U^2) = 0$, i.e., $\mu_{ij} = (h_{ij}^{\gamma} - H\delta_{ij}) = 0$. The corollary obviously holds. When $n < 5$, the corollary implies Theorem 1 in [4].

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四元数射影空间中具有平行平均曲率向量的全实叶的 Pinching 定理

彭慧春¹, 李志波²

(1- 华北电力大学数理系, 北京 102206; 2- 郑州大学数学系, 郑州 450052)

摘 要: 本文利用 Nakagawa 和 Takagi 的计算散度的方法, 求出四元数射影空间上具有平行平均曲率向量的全实黎曼叶状结构 F 的向量场的散度, 并证明了其上的一个 Simons 型的 Pinching 定理。

关键词: 黎曼叶状结构; 四元数射影空间; 平均曲率; 散度; 全脐